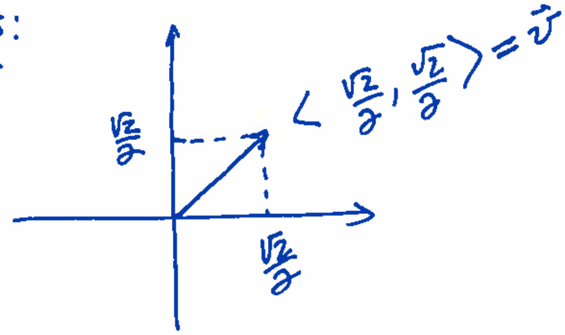
.2 Unit Vectors. Vectors Representations. Prob. # 37

Some definitions. Illustrating examples.

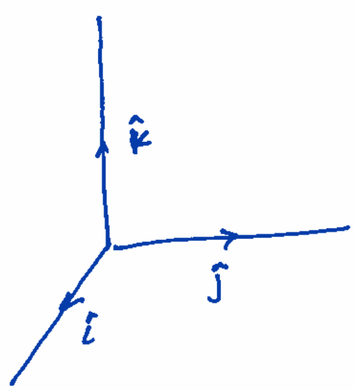
Def.- A "unit vector" is a vector of length 1.

Examples:



$$|\vec{v}| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = 1 .$$

other examples are



$\hat{i} = \langle 1,0,0 \rangle$

$\hat{j} = \langle 0,1,0 \rangle$

$\hat{k} = \langle 0,0,1 \rangle$

Standard basis Vectors.

Given a Vector $\vec{v}$, we can always obtain a unit vector in the same direction of $\vec{v}$

In fact,

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

Show that $\frac{\vec{v}}{|\vec{v}|}$ is a unit vector

Proof. If $\vec{v} = \langle v_1, v_2, v_3 \rangle$

then $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

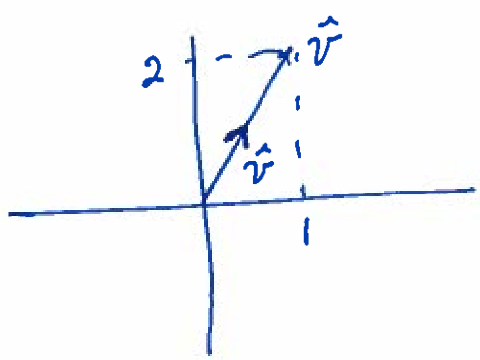
and $\left| \frac{\vec{v}}{|\vec{v}|} \right| = \left| \left\langle \frac{v_1}{\sqrt{v_1^2+v_2^2+v_3^2}}, \frac{v_2}{\sqrt{v_1^2+v_2^2+v_3^2}}, \frac{v_3}{\sqrt{v_1^2+v_2^2+v_3^2}} \right\rangle \right|$

$$= \sqrt{\frac{v_1^2}{v_1^2+v_2^2+v_3^2} \quad \frac{v_2^2}{v_1^2+v_2^2+v_3^2} + \frac{v_3^2}{v_1^2+v_2^2+v_3^2}} = 1.$$

Find a unit vector in the direction of

$\vec{v} = \langle 1, 2 \rangle$

Answ: $|\vec{v}| = \sqrt{1^2+2^2} = \sqrt{5}$, then $\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

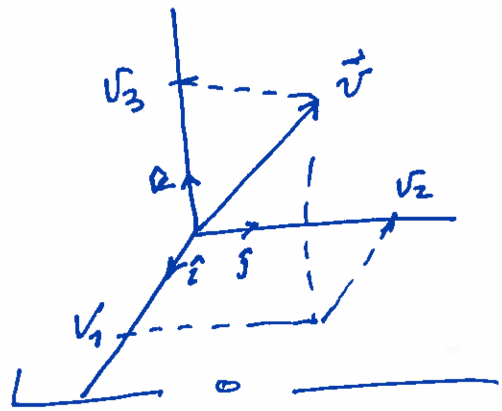


A Vector

(I) $\boxed{\vec{v} = \langle v_1, v_2, v_3 \rangle}$

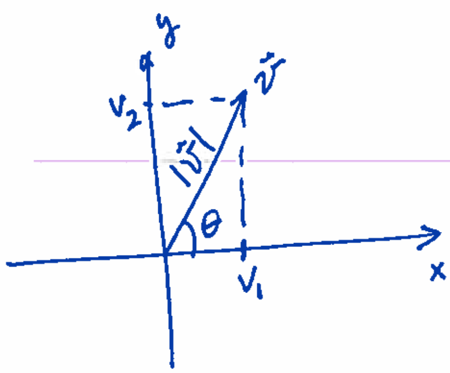
with components v_1 (x-dir.), v_2 (y-dir), and v_3 (z-dir)

Can also be written as



(II) $\boxed{\vec{v} = v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k}}$

Also, notice that for $\vec{v} = \langle v_1, v_2 \rangle = v_1 \hat{i} + v_2 \hat{j}$



$$\cos\theta = \frac{v_1}{|\vec{v}|}, \quad \sin\theta = \frac{v_2}{|\vec{v}|}$$

from elementary trigonometry.

Therefore, $v_1 = |\vec{v}| \cos\theta$, $\quad v_2 = |\vec{v}| \sin\theta$

and

(III) $\boxed{\vec{v} = |\vec{v}| \cos\theta \, \hat{i} + |\vec{v}| \sin\theta \, \hat{j}}$.

Section 12.2 (Exercises).

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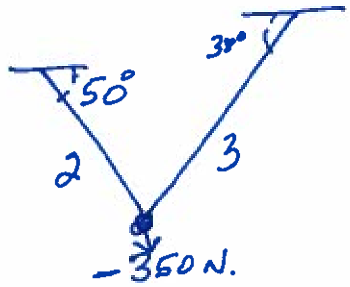

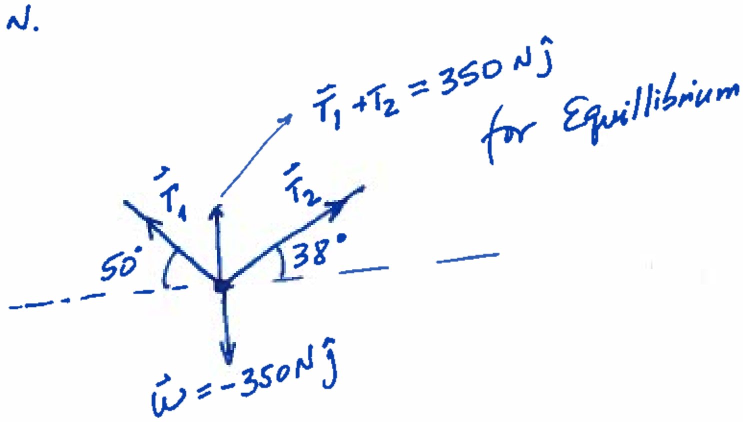

$$\vec{T}_1 = -|\vec{T}_1| \cos 50\,\hat{i} + |\vec{T}_1| \sin 50\,\hat{j}$$

$$\vec{T}_2 = |\vec{T}_2| \cos 38\,\hat{i} + |\vec{T}_2| \sin 38\,\hat{j}$$

$$\vec{T}_1 + \vec{T}_2 = 0\,\hat{i} + 350\,\hat{j}, \text{ then}$$

$$\begin{cases} -|\vec{T}_1| \cos 50 + |\vec{T}_2| \cos 38 = 0 \\ |\vec{T}_1| \sin 50 + |\vec{T}_2| \sin 38 = -350 \end{cases}$$

$$\left[\begin{array}{cc|c} -\cos 50 & \cos 38 & 0 \\ \sin 50 & \sin 38 & +350 \end{array}\right] \sim \left[\begin{array}{ccc} 1 & -\dfrac{\cos 38}{\cos 50} & 0 \\ \sin 50 & \sin 38 & +350 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & -\dfrac{\cos 38}{\cos 50} & 0 \\ 0 & \sin 38 + \cos 38 \dfrac{\sin 50}{\cos 50} & +350 \end{array}\right]$$

$$|\vec{T}_1| = |\vec{T}_2| \frac{\cos 38}{\cos 50} \simeq |\vec{T}_2| \frac{0.788}{0.643} \simeq 1.226 \, |\vec{T}_2|$$

$$|\vec{T}_2| = \frac{+350}{\sin 38 + \cos 38 \tan 50} \simeq \frac{350}{0.616 + 0.788 * 1.192} \simeq 225.066$$

then,

$$|\vec{T}_1| \simeq 1.226 * 225.066 \simeq 275.93$$

Therefore,

$$\vec{T}_1 \simeq -275.93 \cos 50 \,\hat{i} + 275.93 \sin 50 \,\hat{j} \simeq -177.36\hat{i} + 211.37\hat{j}$$

$$\vec{T}_2 \simeq 225.07 \cos 38 \,\hat{i} + 225.07 \sin 38 \,\hat{j} \simeq$$

or

$$\boxed{\vec{T}_2 \simeq 177.36\,\hat{i} + 138.57\,\hat{j}.}$$

$$\boxed{\vec{T}_1 \simeq -177.36\,\hat{i} + 211.37\,\hat{j}}$$